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OSCILLATION PROPERTIES OF TWO-TERM HIGHER-ORDER DIFFERENTIAL EQUATION

ABSTRACT
of the dissertation for the degree of Doctor of Philosophy (PhD) in the specialty 6D060100 – Mathematics

Relevance of the research topic. The dissertation is devoted to the study of the oscillatory properties of a two-term linear differential equation of the fourth and $2n\ \ (n>2)$ order.

The qualitative properties study of differential equations began to appear in the 19th century in the physics and mechanics tasks. Sturm was investigating the behavior of linear differential equation solutions, at first he introduced the concept of oscillatory equation as an equation with the property that any solutions have an infinite number of zeros and also proved his well-known theories of comparison and separation of zeros in 1836.

The oscillatory properties study of differential equation of second-order started by the famous work of Sturm and continues to develop to this day.

The study of the oscillatory properties of a differential equation is a branch of mathematical analysis, and many articles and books had published by this topic. Most of the results refer to second-order equations. Equations of the fourth and higher order have been studied poorly. Only some of the cases considered, when part or all of the coefficients of the equation are power functions. Since, this is due to the lack of simple and effective research methods of second-order equations.

One of the research methods of the oscillatory properties of fourth and higher order differential equations is the transition from an equation to a Hamiltonian system of equations and the application to the resulting system method by Ricatti. However, even here the difficulties arise with the search for a fundamental solution to the Hamiltonian system. Another method is called the “variational principle” or simply the “variational method”. The problems under consideration of the variational methods are reduced to the study of higher order weighted differential inequalities, which the research has not yet been fully completed.

In many works related to the topic of the dissertation, the oscillatory properties of a fourth-order differential equation were researched. Analysis of these works showed that there are many disadvantages for fourth-order equations. They can be divided into two groups. Disadvantages of one of the groups, where only one of the coefficients is an arbitrary function, and the others are power functions, or all the coefficients are power functions. Therefore, due to the impossibility of research of differential equations by the above problems, the coefficients, which are arbitrary functions, various perturbations of the well-known Euler equation are considered.

The second group of disadvantages is that when passing from a higher order differential equation to a system of differential equations, the obtained results are expressed by solutions of this system. In the study of differential equations of the $2n \ (n > 2)$ order, the disadvantages for the equations of the fourth order are repeated. Thus, the description problem of the characteristics of oscillatory and non-oscillatory differential equations of the fourth and $2n \ (n > 2)$ orders in terms of coefficients remains open.

The dissertation work is devoted to the study of the above-mentioned actual issue by the variational method. First, criteria for the fulfillment of a second-order weighted differential inequality and two-sided estimates for its smallest constant is obtained depending on the degree of singularity of the weight functions at infinity.

On the basis of the obtained results the properties of the oscillatory and non-oscillatory properties of the binomial differential equation of the fourth order are comprehensively investigated, and, depending on singularity of the coefficients of the equation the degree at infinity results are given only in terms of the coefficients. The obtained results for a binomial differential equation of the $2n \ (n > 2)$ order are based on the result, which obtained in one case of a weighted differential inequality of the $n \ (n > 4)$ order.

The oscillatory properties of differential equations are studied by scientists from around the world. For example, scientists from Russia, China, America, Japan, Turkey, Saudi Arabia, Egypt, France, Germany, Czech Republic and Hungary.

In Kazakhstan, the study of oscillatory properties of differential equations began in the work of M. Otelbaev and continued in the works of R. Oinarov, L. Kussainova, B. Koshkarova, K. Myrzataeva, M. Aldai, S. Alimagambetova, S. Kudabaeva, Kh. Ramazanova defended their dissertations on this topic.

The purpose of the dissertation research. **The purpose of the study** is to find in terms of the necessary coefficients and sufficient conditions for the oscillatory and non-oscillatory properties at infinity for a two-term differential equation of the fourth and $2n \ (n > 2)$ order with non-negative coefficients.

Tasks of the dissertation research. To achieve this goal, you need to solve the following tasks:

- Obtain necessary and sufficient conditions for the second-order weight differential inequality in terms of weight functions;
- Obtain two-way estimates for the best constant in a second-order weighted differential inequality and calculate the equivalence constants;
- Obtain the necessary, sufficient conditions for the oscillation and non-oscillation of a two-term linear differential equation of the fourth order in terms of coefficients;
- Obtain necessary and sufficient conditions for strong oscillation and strong non-oscillation of a two-term linear differential equation in terms of coefficients;

- Obtain the necessary, sufficient conditions for the oscillation and non-oscillation of a linear differential equation of the $2n$ $(n>2)$ order in terms of coefficients.

Object of the research. The object of research is linear differential equations of the fourth and $2n$ $(n>2)$ order, weighted differential inequalities of the second and n $(n>4)$ order.

Research method. Using the results of the theory of weighted differential inequality are investigated the oscillatory and non-oscillatory properties of the differential equation on the basis of the variational principle.

Scientific novelty of the research. A new condition for the fulfillment of a second-order differential inequality with arbitrary non-negative weights and an estimation of its best constant, new conditions for the oscillatory and non-oscillatory linear differential equation of a fourth and $2n$ $(n>2)$ order with arbitrary non-negative coefficients.

The following results are submitted for defense.

• Necessary and sufficient conditions for the fulfillment of a second-order weight differential inequality in terms of weight functions obtained;

• For the best constant in a weighted second-order differential inequality, two-sided estimates obtained and the equivalence constants were calculated;

• Necessary, sufficient conditions for oscillatory and non-oscillatory properties of a two-term linear differential equation of the fourth order in terms of coefficients;

• Necessary and sufficient conditions for strong oscillation and non-oscillation of a two-term linear differential equation in terms of coefficients;

• Necessary, sufficient conditions for the oscillation and non-oscillation of a linear differential equation of the $2n$ $(n>2)$ order in terms of coefficients.

The theoretical and practical value of the results. The work is theoretical. The results in this work on a weighted differential inequality of the second order are a contribution to the theory of weighted inequalities and can be used in the theory of differential equations and integral operators, in harmonic analysis. The results on the oscillatory and non-oscillatory properties of the fourth and $2n$ $(n>2)$ order two-term linear differential equations are a contribution to the qualitative theory of differential equations and, also can be used in the spectral theory of differential operators.

Personal contribution of the PhD student. The research work presented in the dissertation was carried out with the direct participation of the author. To obtain the condition of oscillation and non-oscillation of a higher-order differential equation, we describe the space under consideration under various conditions and consider the implementation of the differential inequality and calculate the equivalent constants for the best constant of this inequality. On the basis of the variational method, new conditions of oscillation and non-oscillation, strong oscillation and non-oscillation of a higher-order differential equation are obtained. The results were published in the form of scientific articles and scientific reports.

Approbation of the dissertation results. The main results and scientific conclusions of the dissertation researches were discussed at the scientific seminars "Functional Analysis and Its Applications" organized by the Fundamental Mathematics Department of the Eurasian National University named after L. Gumilyov; and also were reported at the traditional international April mathematical conference in honor of the Day of Science Workers of the Republic of Kazakhstan and Workshop "Problems of modeling processes in electrical contacts", Almaty, April 3-5, 2019; the international scientific conference "Theoretical and applied issues of mathematics, mechanics and computer science", Karaganda, June 12-13, 2019; the international conference "Actual problems of analysis, differential equations and algebra" (EMJ-2019), Nur-Sultan, October 16-19, 2019; the international Uzbek and Russian scientific conference "Non-classical equations of mathematical physics and their applications", Tashkent, October 24-26, 2019.

Publications. The main results of the dissertation research were published in 11 papers, including 4 articles in the publications recommended by the Committee for Control in Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan, 1 article in a journal from the Scopus database with a CiteScore percentile of at least 25 and 5 theses in the materials of international scientific conferences.

The volume and structure of the dissertation. The dissertation is written in the Kazakh language, consists of an introduction, the main part of three sections and a conclusion, a list of used literature, including 75 titles. The total volume of the dissertation work is 99 pages.

In the first chapter, the necessary known facts and statements are given and conditions for the fulfillment of a second-order weighted differential inequality are obtained. In the second chapter, the conditions for oscillatory and non-oscillatory properties of a two-weight linear differential equation of the fourth order, a linear and semilinear differential equation, and conditions for strong oscillation and strong non-oscillatory character of a two-weight linear differential equation are obtained. In the third chapter, the conditions for oscillatory and non-oscillatory properties of a linear differential equation of higher order are obtained. In the conclusion, the main conclusions and results of the work are formulated.